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OF SLENDER WINGS AT HYPERSONIC SPEEDS

BY

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LIFT-TO-DRAG RATIOS OF SLENDER WINGS AT HYPERSONIC SPEEDS (*)

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ANGELO MIELE (**)

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SUMMARY

An investigation of the lift-to-drag ratio attainable by a slender, affine wing at hypersonic speeds is presented under the assumptions that the pressure distribution is Newtonian and the skin-friction coefficient is constant. Analytical expressions are derived relating the drag, the lift, and the lift-to-drag ratio to the geometry of the configuration.

The class of flat-top wings whose upper surface is parallel to the free stream is considered. After it is assumed that the chordwise thickness

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distribution is a power law and the spanwise thickness distribution is proportional to some power of the chord distribution, the effect of the thickness ratio and the power law exponents on the lift-to-drag ratio is investigated.

It is shown that a set of values of the thickness ratio and the power law exponents exists which yields a maximum lift-to-drag ratio. Specifically, the optimum thickness ratio is such that the friction drag is one-third of the total drag; the optimum chordwise power law exponent is one, meaning that a linear thickness distribution is the best in the chordwise sense; and the optimum spanwise power law exponent is one, meaning that a thickness distribution proportional to the chord distribution is the best in the spanwise sense. For a friction coefficient $C_f = 10^{-3}$, the maximum attainable lift-to-drag ratio is 5.29.

author

1. INTRODUCTION

In a previous report (Ref. 1), the basic theory of slender, lifting bodies in the hypersonic regime was formulated, and the lift-to-drag ratios attainable by these bodies were determined. The class of flat-top bodies whose longitudinal contours are power laws and whose transversal contours are semielliptical or triangular was considered, and the effects of the thickness ratio, the power law exponent, and the elongation ratio of the cross section on the lift-to-drag ratio were investigated.

It was shown that, if the cross section is triangular, a wing-like configuration rather than a body-like configuration is aerodynamically desirable at hypersonic speeds. Because of this result and owing to the fact that the analytical treatment of wing-like configurations requires assumptions different from those employed for body-like configurations, the analysis of Ref. 1 is extended here to slender, lifting wings.

It is the object of this report to formulate the basic theory of slender, lifting wings and determine the maximum attainable lift-to-drag ratios under the assumption that no constraints are imposed on the configuration. In a practical design, requirements may be imposed on quantities such as the volume, the planform area, the frontal area, and the position of the center of pressure. Therefore, the lift-to-drag ratios calculated here must be regarded as the upper limits to those which can be achieved whenever any combination of constraints is imposed on the wing.

The hypotheses employed are as follows: (a) a plane of symmetry exists between the left-hand and right-hand parts of the wing; (b) no plane of symmetry exists between the upper and lower parts; however, the intersection of these parts is a curve contained in a plane perpendicular to the plane of symmetry, called the reference plane; (c) the wing is slender in both the chordwise and spanwise senses, that is, the squares of both the chordwise and spanwise slopes are small with respect to one; (d) the wing is affine, in the sense that each chordwise section can be generated from the root section by a linear transformation not involving rotation; (e) the free-stream velocity is parallel to the line of intersection of the plane of symmetry and the reference plane; (f) the pressure coefficient is twice the cosine squared of the angle formed by the free-stream velocity and the normal to each surface element; (g) the skin-friction coefficient is constant; and (h) the contribution of the tangential forces to the lift is negligible with respect to the contribution of the normal forces.

2. DRAG AND LIFT

In order to relate the drag and the lift of a wing to its geometry, we consider a Cartesian coordinate system Oxyz defined as follows (Fig. 1): the origin O is the apex of the wing; the x-axis is the intersection of the plane of symmetry and the reference plane, positive toward the trailing edge; the z-axis is contained in the plane of symmetry, perpendicular to the x-axis, and positive downward; and the y-axis is such that the xyz-system is right-handed.

We now denote by $\vec{u}_x$, $\vec{u}_y$, $\vec{u}_z$ the unit vectors of the Cartesian coordinate system, by $\vec{n}$ the unit vector normal to the infinitesimal element of wetted area dS_w, positive outward, and by $\vec{t}$ the unit vector which is tangent to dS_w and is in the direction of the local flow after impact. Since the free-stream velocity is parallel to the x-axis, the drag D and the lift L are given by

$$\begin{aligned} D/q &= \iint_{S_w} \left(-C_p \vec{n} \cdot \vec{u}_x + C_f \vec{t} \cdot \vec{u}_x\right) dS_w \\ L/q &= \iint_{S_w} \left(C_p \vec{n} \cdot \vec{u}_z - C_f \vec{t} \cdot \vec{u}_z\right) dS_w \end{aligned} \tag{1}$$

where q is the free-stream dynamic pressure, C_p the pressure coefficient, and C_f the skin-friction coefficient.

If every surface element "sees" the flow, the distribution of pressure coefficients is given by (Ref. 2, Chapter 22)

$$C_p = 2\left(\vec{n} \cdot \vec{u}_x\right)^2 \tag{2}$$

Next, the geometric relationship

$$dS_w = \mp\left(\vec{n} \cdot \vec{u}_z\right)^{-1} dx\, dy \tag{3}$$

is introduced, where the upper sign is valid for the upper surface and the lower sign, for the lower surface. With this understanding, the aerodynamic forces can be rewritten in the form

$$\begin{aligned} D/q &= \iint_{S_1+S_2} \left(\mp\vec{n} \cdot \vec{u}_z\right)^{-1} \left[-2\left(\vec{n} \cdot \vec{u}_x\right)^3 + C_f \vec{t} \cdot \vec{u}_x\right] dx\, dy \\ L/q &= \iint_{S_1+S_2} \left(\mp\vec{n} \cdot \vec{u}_z\right)^{-1} \left[2\left(\vec{n} \cdot \vec{u}_x\right)^2 \vec{n} \cdot \vec{u}_z - C_f \vec{t} \cdot \vec{u}_z\right] dx\, dy \end{aligned} \tag{4}$$

where S_1 and S_2 denote the projections of the upper and lower surfaces on the reference plane $z = 0$, each of which is equal to the planform area S. After the normal and tangent vectors are expressed by

$$\vec{n} = \alpha \vec{u}_x + \beta \vec{u}_y + \gamma \vec{u}_z \quad \begin{cases} \gamma \le 0 & \text{over } S_1 \\ \gamma \ge 0 & \text{over } S_2 \end{cases}$$

$$\vec{t} = a \vec{u}_x + b \vec{u}_y + c \vec{u}_z , \quad a \ge 0 \tag{5}$$

Eqs. (4) become

$$D/q = \iint_{S_1 + S_2} (\mp 1/\gamma) \left(-2\alpha^3 + C_f a \right) dx\,dy$$

$$L/q = \iint_{S_1 + S_2} (\mp 1/\gamma) \left(2\alpha^2 \gamma - C_f c \right) dx\,dy \tag{6}$$

If the geometry of the wing is described by the equation

$$f(x, y, z) = 0 \tag{7}$$

the normal unit vector and the gradient of the function f are parallel with the implication that

$$\alpha = f_x/g , \qquad \beta = f_y/g , \qquad \gamma = f_z/g \tag{8}$$

where

$$g = \sqrt{f_x^2 + f_y^2 + f_z^2} \tag{9}$$

Since the tangent vector is of unit modulus, perpendicular to $\vec{n}$, and coplanar with $\vec{n}$ and $\vec{u}_x$, the equations determining the components a, b, c are written as

$$\vec{t} \cdot \vec{t} = 1 , \qquad \vec{t} \cdot \vec{n} = 0 , \qquad \vec{t} \times \vec{n} \cdot \vec{u}_x = 0 \tag{10}$$

and, in explicit form, become

$$\begin{aligned} a^2 + b^2 + c^2 &= 1 \\ \alpha a + \beta b + \gamma c &= 0 \\ \gamma b - \beta c &= 0 \end{aligned} \tag{11}$$

These equations are solved by

$$a = \sqrt{1 - \alpha^2} , \qquad b = -\alpha\beta / \sqrt{1 - \alpha^2} , \qquad c = -\alpha\gamma / \sqrt{1 - \alpha^2} \tag{12}$$

with the implication that

$$a = h/g\,, \qquad b = -f_x f_y/gh\,, \qquad c = -f_x f_z/gh \tag{13}$$

where

$$h = \sqrt{f_y^2 + f_z^2} \tag{14}$$

The next step consists of combining Eqs. (6), (8), and (13) to obtain the relationships

$$\begin{aligned} D/q &= \iint_{S_1 + S_2} (\mp 1/f_z) \left(-2 f_x^3/g^2 + C_f h\right) dx\,dy \\ L/q &= \iint_{S_1 + S_2} (\mp f_x) \left(2 f_x/g^2 + C_f/h\right) dx\,dy \end{aligned} \tag{15}$$

If the geometry of the wing is expressed in the form

$$f(x, y, z) \equiv \mp z \pm z(x, y) = 0 \tag{16}$$

the following relationships hold:

$$f_x = \pm z_x\,, \qquad f_y = \pm z_y\,, \qquad f_z = \mp 1 \tag{17}$$

and imply that

$$g = \sqrt{1 + z_x^2 + z_y^2}, \qquad h = \sqrt{1 + z_y^2} \tag{18}$$

Consequently, Eqs. (15) can be rewritten as

$$\begin{aligned} D/q &= \iint_{S_1 + S_2} \left(\mp 2 z_x^3/g^2 + C_f h\right) dx\,dy \\ L/q &= \iint_{S_1 + S_2} z_x \left(\mp 2 z_x/g^2 - C_f/h\right) dx\,dy \end{aligned} \tag{19}$$

and simplify to

$$\begin{aligned} D/q &= \iint_{S_1 + S_2} \left(\mp 2 z_x^3/g^2 + C_f h\right) dx\,dy \\ L/q &= \iint_{S_1 + S_2} \left(\mp 2 z_x^2/g^2\right) dx\,dy \end{aligned} \tag{20}$$

if the contribution of the tangential forces to the lift is negligible with respect to the contribution of the normal forces, that is, if

$$\mp C_f\, g^2/2 z_x h << 1 \tag{21}$$

almost everywhere.

2.1. Slender Wing. Now, we consider the contours obtained by intersecting the surface (16) with either a plane $y = \text{Const}$ or a plane $x = \text{Const}$, and observe that the slopes of these contours are given by z_x and z_y, respectively. If the wing is slender in both the chordwise and spanwise senses, the squares of these slopes are small with respect to one, that is,

$$z_x^2 << 1 , \qquad z_y^2 << 1 \tag{22}$$

almost everywhere, so that

$$g = h = 1 \tag{23}$$

With this understanding, Eqs. (20) simplify to

$$\begin{aligned} D/q &= \iint_{S_1 + S_2} \left(\mp 2 z_x^3 + C_f \right) dx\, dy \\ L/q &= \iint_{S_1 + S_2} \left(\mp 2 z_x^2 \right) dx\, dy \end{aligned} \tag{24}$$

2.2. Flat-Top Wing. If the upper and lower parts of the wing are symmetric with respect to the reference plane, the integral (24-2) implies that the lift is zero. However, lift can be produced if the upper and lower parts are not symmetric. Since the contribution of the upper part to the lift is negative and the contribution of the lower part is positive, the lift can be increased by making the volume above the reference plane as small as possible and, preferably, zero. If this is done, the pressure drag is decreased. Therefore, a flat-top wing (Fig. 2) is naturally suited to produce high lift-to-drag ratios in the hypersonic regime. Mathematically speaking, this wing has the property that

$$z = 0 \tag{25}$$

for the upper surface and, as a consequence, Eqs. (24) become

$$\begin{aligned} D/q &= 2 \int\int_S \left(z_x^3 + C_f\right) dxdy \\ L/q &= 2 \int\int_S z_x^2 \, dxdy \end{aligned} \tag{26}$$

where S denotes the planform area and z(x, y) is the function describing the lower surface. Now, let the geometry of the planform and the thickness distribution on the periphery of the planform be expressed as

$$\begin{aligned} &\underline{\text{Leading edge}} \qquad x = m(y)\,, \qquad z = 0 \\ &\underline{\text{Trailing edge}} \qquad x = n(y)\,, \qquad z = t(y) \end{aligned} \tag{27}$$

In the light of the symmetry property, the aerodynamic forces become

$$\begin{aligned} D/q &= 4\int_0^{b/2}\int_{m(y)}^{n(y)} z_x^{\,3}\,dy\,dx + 4\,C_f\int_0^{b/2} c(y)\,dy \\ L/q &= 4\int_0^{b/2}\int_{m(y)}^{n(y)} z_x^{\,2}\,dy\,dx \end{aligned} \tag{28}$$

where b denotes the wing span and

$$c(y) = n(y) - m(y) \tag{29}$$

the chord distribution.

2.3. Affine Wing. Next, we focus our attention on the class of wings such that any chordwise contour can be generated from the root contour by means of a linear transformation not involving rotation. The geometry of the lower surface of these affine wings is given by

$$z = t(0)\,A(\xi)\,B(\eta) \tag{30}$$

where ξ denotes a nondimensional chordwise coordinate, η a nondimensional spanwise coordinate, $A(\xi)$ a function describing the chordwise thickness distribution, and $B(\eta) = t(b\eta/2)/t(0)$ a function describing the spanwise thickness distribution. Specifically, the coordinates ξ and η are defined as

$$\xi = \frac{x - m(y)}{c(y)} \qquad \eta = \frac{y}{b/2} \tag{31}$$

and, hence, vary between the limits 0 and 1. Furthermore, the functions $A(\xi)$ and $B(\eta)$ are chosen in such a way that

$$A(0) = 0\,, \qquad A(1) = 1\,; \qquad B(0) = 1 \tag{32}$$

We now observe that

$$z_x = \tau \dot{A} B/C \tag{33}$$

where $\tau = t(0)/c(0)$ denotes the thickness ratio of the root airfoil, $\dot{A}$ denotes the derivative $dA/d\xi$, and where $C(\eta) = c(b\eta/2)/c(0)$ is a function describing the spanwise chord distribution such that

$$C(0) = 1 \tag{34}$$

With this understanding, the drag and the lift can be rewritten as

$$\begin{aligned} D/q\,bc(0) &= \tau^3 I_1 J_1 + C_f I_2 J_2 \\ L/q\,bc(0) &= \tau^2 I_3 J_3 \end{aligned} \tag{35}$$

where I_1, I_2, I_3 denote the following integrals depending on the chordwise contour:

$$\begin{aligned} I_1 &= \int_0^1 \dot{A}^3\, d\xi \\ I_2 &= 1 \\ I_3 &= \int_0^1 \dot{A}^2\, d\xi \end{aligned} \tag{36}$$

and J_1, J_2, J_3 denote the following integrals depending on the spanwise contour and the chord distribution:

$$
\begin{aligned}
J_1 &= 2\int_0^1 (B^3/C^2)\, d\eta \\
J_2 &= 2\int_0^1 C\, d\eta \\
J_3 &= 2\int_0^1 (B^2/C)\, d\eta
\end{aligned}
\tag{37}
$$

3. LIFT-TO-DRAG RATIO

From the previous discussion, it appears that -- if the root chord $c(0)$, the span b, the thickness ratio τ, the chordwise contour $A(\xi)$, the spanwise contour $B(\eta)$, and the chord distribution $C(\eta)$ are given -- the drag and the lift can be evaluated from Eqs. (35) through (37). Once those quantities are known, one can determine the aerodynamic efficiency or lift-to-drag ratio

$$E = L/D \tag{38}$$

which, in the light of Eqs. (35), can be written as

$$E = \tau^2 I_3 J_3 / (\tau^3 I_1 J_1 + C_f I_2 J_2) \tag{39}$$

3.1. Optimum Thickness Ratio. We now assume that the chordwise contour $A(\xi)$, the spanwise contour $B(\eta)$, and the chord distribution $C(\eta)$ are arbitrarily prescribed, which means that the quantities I_1, I_2, I_3 and J_1, J_2, J_3 are known a priori. Then, we study the effect of the thickness ratio on the lift-to-drag ratio (39).

Clearly, the lift-to-drag ratio is a maximum when the thickness ratio satisfies the relationship

$$\tau = \sqrt[3]{2\,C_f\, I_2 J_2 / I_1 J_1} \tag{40}$$

which means that the friction drag is one-third of the total drag. The associated lift-to-drag ratio is given by

$$E = (I_3 J_3 / 3) \sqrt[3]{4/C_f\,(I_1 J_1)^2\, I_2 J_2} \tag{41}$$

3.2. Optimum Chordwise Contour. Next, we consider wings optimized with respect to the thickness ratio and assume that the spanwise contour $B(\eta)$ and the chord distribution $C(\eta)$ are arbitrarily prescribed, which means that the quantities J_1, J_2, J_3 are known a priori. Then, we consider a one-parameter family of chordwise contours having the form

$$A = A(\xi, n) \tag{42}$$

and study the effect of the parameter n on the lift-to-drag ratio (41). Since this quantity depends on n through the functions I_1, I_2, I_3, a maximum occurs when the following relationship is satisfied:

$$2\dot{I}_1/I_1 + \dot{I}_2/I_2 - 3\dot{I}_3/I_3 = 0 \tag{43}$$

with the dot sign denoting a derivative with respect to n. For the power law contours

$$A = \xi^n \tag{44}$$

the functions I_1, I_2, I_3 take the values

$$\begin{aligned} I_1 &= n^3/(3n - 2) \\ I_2 &= 1 \\ I_3 &= n^2/(2n - 1) \end{aligned} \tag{45}$$

which hold for $n > 2/3$ only (the pressure drag cannot be negative). Since their derivatives are given by

$$\begin{aligned} \dot{I}_1 &= 6n^2(n - 1)/(3n - 2)^2 \\ \dot{I}_2 &= 0 \\ \dot{I}_3 &= 2n(n - 1)/(2n - 1)^2 \end{aligned} \tag{46}$$

the relationship (43) is solved by

$$n = 1 \tag{47}$$

which means that the optimum chordwise contour is a straight line. With this understanding, the thickness ratio (40) and the lift-to-drag ratio (41) become

$$\tau = \sqrt[3]{2 C_f J_2 / J_1}\ , \qquad E = (J_3/3)\ \sqrt[3]{4/C_f {J_1}^2 J_2} \tag{48}$$

3.3. Optimum Spanwise Contour. Finally, we consider configurations which are optimized with respect to the thickness ratio and the chordwise contour $A(\xi)$. We assume that the chord distribution $C(\eta)$ is arbitrarily given, consider a one-parameter family of spanwise contours having the form

$$B = B(\eta, m) \tag{49}$$

and study the effect of the parameter m on the lift-to-drag ratio (48-2). Since this quantity depends on m through the functions J_1 , J_2 , and J_3 , a maximum occurs when the following relationship is satisfied:

$$2\dot{J}_1/J_1 + \dot{J}_2/J_2 - 3\dot{J}_3/J_3 = 0 \tag{50}$$

with the dot sign denoting a derivative with respect to the parameter m. If the spanwise thickness distribution is proportional to the m-th power of the chord distribution, that is, if

$$B = C^m(\eta) \tag{51}$$

the functions J_1, J_2, J_3 take the values

$$\begin{aligned} J_1 &= 2\int_0^1 C^{3m-2}\, d\eta \\ J_2 &= 2\int_0^1 C\, d\eta \\ J_3 &= 2\int_0^1 C^{2m-1}\, d\eta \end{aligned} \tag{52}$$

Since their derivatives are given by

$$\begin{aligned} \dot{J}_1 &= 6\int_0^1 C^{3m-2} \log C\, d\eta \\ \dot{J}_2 &= 0 \\ \dot{J}_3 &= 4\int_0^1 C^{2m-1} \log C\, d\eta \end{aligned} \tag{53}$$

Eq. (50) is solved by

$$m = 1 \tag{54}$$

for every chord distribution $C(\eta)$. Therefore, the optimum wing is such that the spanwise thickness distribution and the chord distribution are proportional to one another. Regardless of the chord distribution, the optimum thickness ratio (48-1) and the associated lift-to-drag ratio (48-2) become

$$\tau = \sqrt[3]{2\,C_f}\ , \qquad E = 2/3\ \sqrt[3]{2\,C_f} \tag{55}$$

4. DISCUSSION AND CONCLUSIONS

In the previous sections, an analytical derivation of the drag, the lift, and the lift-to-drag ratio of slender, affine wings flying at hypersonic speeds is presented under the assumptions that the pressure distribution is Newtonian and the skin-friction coefficient is constant. Particular attention is devoted to the class of flat-top wings whose upper surface is parallel to the free stream.

It is shown that a value of the thickness ratio exists which maximizes the lift-to-drag ratio; this particular value is such that the friction drag is one-third of the total drag. The subsequent optimization of the chordwise contour indicates that a linear thickness distribution is the best in the class of power law contours. Finally, the effect of the spanwise thickness distribution on the lift-to-drag ratio is investigated under the assumption that the chord distribution is arbitrarily given; it is shown that a spanwise thickness distribution proportional to the chord distribution is the best in the class of power-law relationships between these distributions.

The relevant results are summarized in Table 1 in which the assumption $C_f = 10^{-3}$ is employed. In closing, the following comments are pertinent:

(a) The main drawback of the slender wings considered here is the severe heat transfer occurring at the lines of intersection between the surfaces composing the vehicle. Consequently, the present sharp-edge configurations must be replaced by faired configurations in which the transition from one surface to another occurs with a finite curvature. If this is done, lift-to-drag ratios smaller than those predicted here are to be expected.

(b) To design a practical hypersonic vehicle, the present idealized configurations are to be modified by additional elements, such as control surfaces. Hence, a further reduction in the lift-to-drag ratio is to be expected.

(c) The cumulative detrimental effect of the considerations (a) and (b) can be offset to some degree by inclining the upper surface at a negative angle with respect to the flow, that is, by taking advantage of the added lift produced by the flow expansion. If this is done, it is probable that a lift-to-drag ratio in the neighborhood of 4 to 5 can be achieved in practice. This value is sufficiently high to encourage further studies of hypersonic cruise vehicles, suborbital vehicles, and vehicles for maneuverable reentry from outer space.

TABLE 1

OPTIMUM FLAT-TOP WINGS

τ	0.126
n	1.000
m	1.000
E	5.29

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LIST OF CAPTIONS

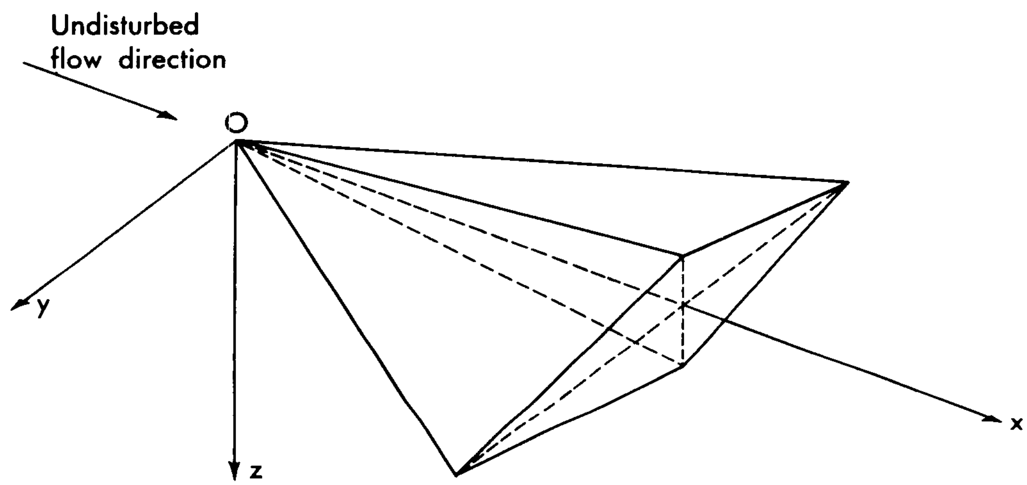

Fig. 1 Coordinate system.

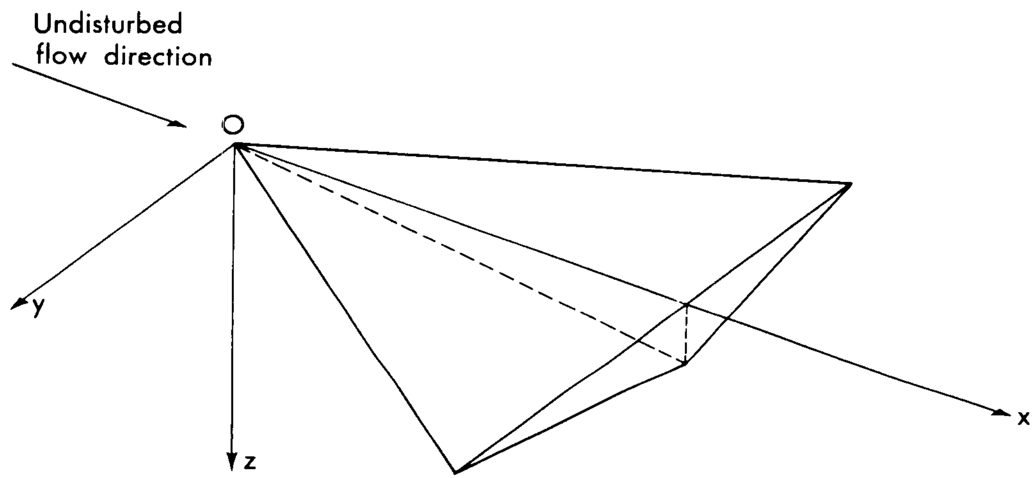

Fig. 2 Flat-top wing